*Korean J. of Chem. Eng.*, 9(3), 117-127 (1992)

# SYNTHESIS OF HEAT EXCHANGER NETWORKS WITH MINIMUM NUMBER OF UNITS FOR PINCHED PROBLEMS

**In-Beum Lee**

Automation Research Center, Pohang Institute of Science and Technology,
P.O. Box 125, Pohang 790-600, Korea
*(Received 16 January 1992 • accepted 11 May 1992)*

**Abstract** – An algorithmic-evolutionary synthesis procedure is studied for generating the maximum energy recovery (MER) and minimum number of units (MNU) networks with the goal of achieving the global optimum network under pinch points. For pinched problems, sufficient conditions are proposed for determining the minimum number of units. These sufficient conditions, together with heuristic matching rules, are used to generate an initial feasible composite MNU/MER network. A split-merge network structure is introduced in order not to violate the prescribed minimum approach temperature. This initial network is successively evolved to obtain improved networks by limited heat load redistribution resulting from the pinch point. The properties and limitations of the constructions and procedures are established and the effectiveness of the heuristic procedure is illustrated with literature test problems.

## INTRODUCTION

Lee and Reklaitis [1] reported on an evolutionary procedure for generating improved heat exchanger networks which feature the maximum energy recovery (MER) and the minimum number of heat exchanger units (MNU) for unpinched problems. In the present work, ways of extending this approach to pinched problems are studied.

In the presence of a pinch temperature, it is well known that the heat exchanger network synthesis problem can be decomposed into two sub-problems [2]: the portion above the pinch (AP) where external heating is required and the portion below the pinch (BP) where external cooling is needed. Given this natural decomposition, one approach to synthesizing pinched networks is to employ existing methods, including the methodology suggested in [1], to generate networks for each of these separate portions which for specified minimum approach temperature $\Delta T_m$, feature both MER and MNU [3-5]. However, the combination of these independently synthesized sub-networks into a single composite network in general leads to more units than the overall minimum required [3]. Thus MER is assured but true MNU sometimes is not achieved. In this case, as proposed in [6], MNU can be maintained by allowing heat transfer across the pinch, thus increasing utility usage and violating MER. The conventional approach to pinched problems is thus to undertake a trade-off study between the goals of MNU (which represents the capital cost) and MER (which reflects utility operating costs) [2].

In some instances the apparently incompatible goals of MNU and MER can be met by relaxation of the minimum approach temperature restriction. This is not unreasonable since in practice the $\Delta T_m$ restriction is a soft constraint. With a lower $\Delta T_m$ it may be possible to synthesize a feasible MNU/MER network but this generally will require some larger units and thus increase the capital cost. Moreover, in some cases it is impossible to find a feasible MNU/MER network even though $\Delta T_m$ is reduced to zero. Thus, the stratagem of reducing $\Delta T_m$ cannot be viewed as a general method but rather as an option which should be considered as one element of a composite synthesis approach.

Finally, Wood et al. [7] proposed a quick method to achieve the MNU target under pinch conditions by using a novel arrangement of stream splitting, mixing, and exchanger by-passing. However, the proposed approach is only described qualitatively in terms of some examples without defining quantitative procedure for determining the split and by-pass ratios.

In this paper we combine elements of the above devices to construct feasible MNU/MER networks for pinched problems. We begin with a brief discussion of an improved formula for predicting MNU under the pinch. Next, the use of splitting and determination of split ratios at the pinch are considered. We continue with discussion of devices for constructing improved networks and conclude with a presentation of applications of the combined approach.

## PREDICTION OF MINIMUM NUMBER OF UNITS UNDER PINCH

For heat exchanger networks, pinched or unpinched, the theoretical (or quasi) minimum number of units ($N_{min}$) can be expressed as follows [8].

$$N_{min}=N_{source}+N_{sink}-1=N_{streams}-1 \quad (1)$$

where $N_{source}$ and $N_{sink}$ is the number of source and sink stream, respectively. The source streams include hot streams and steam while the sink streams include cold streams and cooling water, if utilities are required.

However, for the pinched problem, the following feasibility criteria should be satisfied [2].

$$\text{AP: } c_h \le c_c,\ n_h \le n_c \quad (2)$$

$$\text{BP: } c_h \ge c_c,\ n_h \ge n_c \quad (3)$$

where c is a heat capacity flow rate and n is the number of streams.

If these criteria are not satisfied in the pinched problem, streams should be split to increase the number of streams and reduce the heat capacity flow rate. These splits will in general result in must-matches which can be defined at the pinch. Must-matches obtained under these conditions [Eq. (2) and (3)] are called rule-10 matches, to reflect the addition to the rules defined in the published paper [1]. Must-matches obtained at the pinch from the above feasibility criteria are demonstrated in the following Illustration 1.

**Illustration 1**

For the problem shown in Table 1 [9], pre-analysis results are obtained using conventional targeting methods and must-matches are obtained using first the rules presented in the published paper [1].

$$-H=100.32(\text{kW}),\ C=391.384(\text{kW}),\ T^*=217-227\ (℃)$$

where H and C are the minimum heating and cooling requirements, respectively and T* is the pinch temperature.

For brevity, we use the notation X-Y to indicate a match between a hot stream X and a cold stream Y. For convenience, Rule 1 and 5 for must-matches

**Table 1. Stream data for illustration 1**

| Stream | $T_i$(℃) | $T_t$(℃) | c[kW/℃] | HC[kW] |
|---|---|---|---|---|
| H1 | 160 | 110 | 7.032 | 351.600 |
| H2 | 249 | 138 | 8.440 | 936.840 |
| H3 | 227 | 106 | 11.816 | 1429.936 |
| H4 | 271 | 146 | 7.000 | 875.000 |
| C1 | 96 | 160 | 9.144 | 585.216 |
| C2 | 116 | 217 | 7.296 | 736.896 |
| C3 | 140 | 250 | 18.000 | 1980.000 |

$\Delta T_m=10$℃
H=100.32 kW
C=391.384 kW
T*=217-227℃

are rewritten [1].

Rule 1: If only one hot (cold) stream exists, all the cold (hot) streams must be matched with that stream.

Rule 5: If $T_{tHi}$ and $(T_{icj}+\Delta T_m)$ of the coldest stream are same and $c_{Hi}$ is larger than $c_{cj}$, then stream $H_i$ must be matched with cooling water.

Then must-matches are found as follows:

AP: S-C3 (rule 1), H4-C3 (rule 1 or 10), H2-C3 (rule 1 or 10). To satisfy Eq. (2) C3 should be split into two streams.

BP: H3-C3 (rule 10), H3-C2 (rule 10), H3-W (rule 5), H4-C3 (from AP), H2-C3 (from AP). Here we have six match options because of three hot (H2, H3, H4) and two cold (C2, C3) streams at the pinch. But since H4-C3 and H2-C3 exist already in AP, H3-C2 and H3-C3 are chosen to keep the number of units to as few as possible. To satisfy Eq. (3) C3 and H2 are split into three and two streams, respectively.

To predict the minimum number of units for pinched problems, the hot and cold streams are classified into the two following groups:

Group A

Streams whose temperature ranges enclose the pinch, which belong to both AP and BP.

Group B

Streams which do not belong to Group A and thus appear in either AP or BP.

Clearly, any hot or cold stream must belong to one or the other of the above two groups. Thus

$$N_{strm}=N_A+N_B \quad (4)$$

where $N_{strm}$ is the number of stream and $N_A$ and $N_B$ are number of streams belonging to Group A and B, respectively.

When pinched problems are solved by dividing the network into two unpinched sub-problems at the pinch and svnthesizing the associated sub-networks independently, the minimum number of units in the network will be [2]

$$N_{min,pinched} = 2N_A + N_B - 2 = N_A - 1 + N_{strm} - 1 = N_A - 1 + N_{min} \quad (5)$$

If $N_A$ is one (only one cold stream includes the pinch point in its temperature range), the minimum number of units for the pinched problem is exactly the same as that of an unpinched problem. Therefore, this class of problems, problem 4SP1 is an example, can be treated as unpinched problems by synthesizing the AP subnetwork (match between steam and the single cold stream) and the BP subnetwork independently [3].

If $N_A$ is greater than one, the theoretical minimum number of units in a network cannot in general be achieved by superimposing the sub-networks. But, under the split-merge method [7] which enables heat transfer across the pinch, a pair of hot and cold streams belonging to group A can be matched by allowing maximum heat transfer, thus treating those streams as whole streams. In other words, the matched streams are not divided into two sub-streams for separate matching within each subnetwork. Then Eq. (5) becomes

$$N_{min,pinched} = N_{min} + N_A - 1 - \min(N_h, N_c)_A \quad (6)$$

**Proposition 1. Matching Rule (Cold Stream Splitting)**

For pinched problems, the following two conditions should be satisfied as sufficient conditions for reducing the number of units in the network via cold stream splitting. For any two streams of Group A selected to be matched, the conditions are described as follows.

1. For a hot stream whose heat capacity flow rate is smaller than that of the cold stream, the hot stream target temperature should be higher by at least $\Delta T_m$ than the inlet temperature of the cold stream.

2. The heat content of the cold stream should be larger than that of the hot stream in AP.

To prove Proposition 1, if a chosen hot stream has a lower target temperature than the inlet temperature of the corresponding cold stream in the BP zone, the hot stream will have remaining energy of $c_h$ ($T_{ci} + \Delta T_m - T_{th}$) even if the maximum allowable heat is transferred between those streams. Since two streams are still left unmatched after matching, the resulting synthesized network, in general, cannot have MNU. Furthermore, to eliminate that hot stream after a match in AP, the heat content of the cold stream must be greater than that of the hot stream. Otherwise, the hot stream requires an additional match, resulting in MNU violation.

Since these conditions enable the last term of Eq. (6) to be effective, they can be used as matching rules for generating initial networks. If these conditions are not met, the number of units in the initial network cannot be reduced. It should be noted that the converse case to the hypothesis of Proposition 1, namely, the cold stream has a smaller heat capacity flow rate than the hot one is inapplicable because stream splitting cannot raise the inlet temperature of the hot stream. If $N_{hA}$ and $N_{cA}$ are not equal, then the theoretical MNU cannot be satisfied and the difference between those two values represents extra units.

To reduce the number of units further, we can use such a special case (which can be called a perfect or isolated match) that the heat contents of the selected hot and cold streams are exactly the same and the match between the two does not violate $\Delta T_m$ constraints. Even though Douglas [10] classified it as an independent problem, the number of units in a network is reduced by one if any perfect match is found.

**Proposition 2. Matching Rule**

Match streams so as to obtain perfect matches, that is, so that the heat contents of both the hot and the cold stream are exactly the same in either AP or BP.

For the proof of Proposition 2, under the hypothesis, for each unit either a source or a sink stream will be eliminated from the unmatched streams. Since the heat exchanged in that unit is equal to the value of the heat contents of the source or the sink streams. Thus for every match, the number of remaining unmatched streams is reduced by one. However, since every perfect matches leave two streams matched at the same time, we can remove extra streams from the remaining unmatched streams. Therefore, the number of perfect matches can be also subtracted from the number of streams, reducing the number of units by that number.

Let $N_{pm,pinched}$ be the number of perfect matches. Then

$$N_{pm,pinched} = \{N \mid N(HC_h = HC_c)\} \quad (7)$$

where HC represents heat content of stream.

Since a perfect match reduces the number of units in a network by one, the number of perfect matches is subtracted from the number of units. From Eq. (6),

$$N_{min,pinched} = N_{min} + N_A - 1 - \min(N_h, N_c)_A - N_{pm,pinched} \quad (8)$$

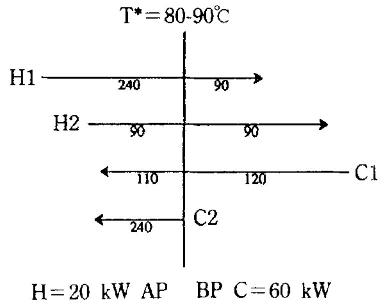
**Illustration 2**

The problem data for this illustration are given in Table 2 [5], including the enthalpy of each stream. We have 6 streams including steam and cooling water,

**Table 2. Stream data for illustration 2 and example problem 1**

| Stream | $T_i$(℃) | $T_t$(℃) | c(kw/℃) | HC(kW) |
|---|---|---|---|---|
| H1 | 170 | 60 | 3.0 | 330. |
| H2 | 150 | 30 | 1.5 | 180. |
| C1 | 20 | 135 | 2.0 | 230. |
| C2 | 80 | 140 | 4.0 | 240. |

$\Delta T_m = 10$℃




three of which belong to group A (H1, H2, C1). Since the heat contents of streams H1 and C2 in AP are exactly the same, the predicted minimum number of units is 6 (=6+3−1−1−1).

When a network is synthesized, the split-merge method requires stream splitting, which in general increases the number of streams and thus units. In order that MNU should not be changed as a result of splitting, every branch of a split stream except one must undergo a perfect match. Thus, to synthesize an MNU network, the following rule for stream splitting can be enunciated.

**Proposition 3. Splitting Rule**

If the cold stream is to be split, then all but one of the split portions should undergo perfect matches with the corresponding hot streams.

To prove Proposition 3, if a split portion of a stream is not matched perfectly, then after the contact, there will be one exchanger and two residuals. Since the number of streams is the same as before, the match will result in a network with one extra unit.

The choice of streams to be matched must be made primarily from the must-matches at the pinch in both sub-problems and must satisfy the sufficient conditions of Propositions 1 and 2. Note that the selected stream with the smaller heat capacity flow rate meets its target after it is matched under Proposition 1 since it is matched perfectly with a split portion of the opposite stream.

## MNU NETWORK

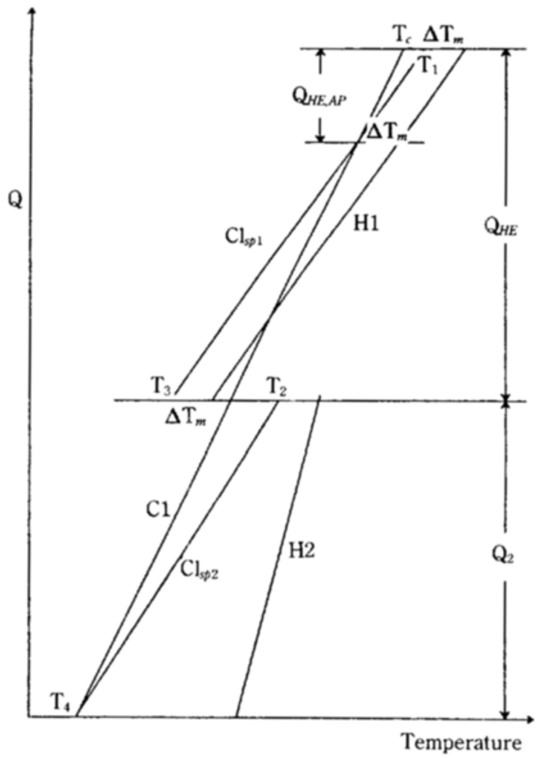



**Fig. 1. Temperature profile with stream splitting.**

For pinched problems, two MNU/MER synthesis matrices can be obtained from the unpinched sub-problems above and below the pinch. In this case, after the minimum heating and cooling requirements are computed for the whole problem, the H/H rule (match between the hottest stream and the cold stream with the highest target temperature) for choosing streams to be matched and the sufficient condition of allowing the maximum heat transfer in a chosen match can be applied to obtain an initial subnetwork for each sub-problem [1]. The sum of the number of units in the two sub-networks usually violates MNU because two identical matches or totally different matches involving the same streams will occur in both sub-networks. This violation of MNU occurs when the number of streams whose temperature ranges enclose the pinch point is greater than one. Furthermore if the heat loads of the identical matches are added to obtain the minimum number of units, the resulting combined units will violate the prescribed $\Delta T_m$. Thus it is necessary to develop a synthesis procedure for obtainng a feasible network from the combined network. We propose to achieve this by splitting and merging

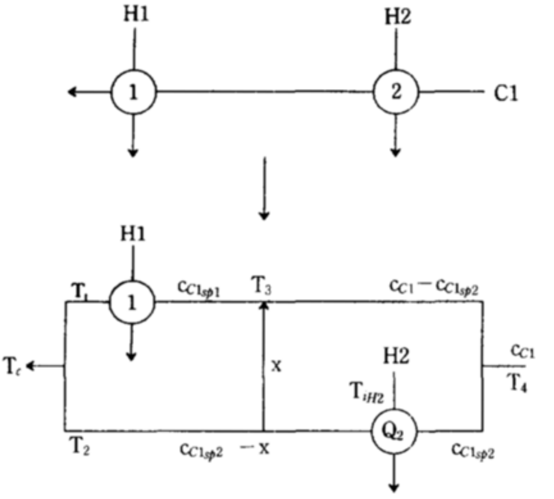



**Fig. 2. Network configuration after introducing stream splitting.**

streams.

Since the AP and BP synthesis matrices cannot be added without violating $\Delta T_m$, we should split a stream to correct this violation. The basic idea is easily represented (the amount of heat exchanged above the pinch) by Fig. 1. For MNU, $Q_{HE}$ (the amount of heat exchanged) must be exchanged but to avoid violating $\Delta T_m$ only $Q_{HE,AP}$ is allowed. If the straight line representing the stream C1 with the larger capacity flow rate is rotated at the pinch point so that the lines corresponding to the two streams (H1 and $C1_{sp1}$) are parallel, $Q_{HE}$ is allowed without violating the prescribed $\Delta T_m$ during the heat exchange between the two streams. Since the slopes (flow rates) of two lines (streams) are exactly the same, the flow rate of C1 must be changed by splitting and the minimum temperature approach will occur over the whole temperature range of the match of H1 and $C1_{sp1}$. The remaining problem is then how to determine the split ratios to satisfy the associated heat balance.

The network configuration of Fig. 1 with split streams is shown in Fig. 2. This network must satisfy the following heat balance equation.

$$c_{C1_{sp1}}T_1+(c_{C1_{sp2}}-x)T_2=c_{C1}T_c \quad (9)$$

or

$$(c_{C1}-c_{C1_{sp2}})T_4+xT_2=c_{C1_{sp1}}T_3 \quad (10)$$

where T and c are temperature and heat capacity flow rate shown in Fig. 2 and x is the split ratio.

Since these two equations are identical, the stream split ratios can be determined from Eq. (9) as follows.

(1) Set the flow rate of $C1_{sp1}$ equal to that of H1

$$c_{C1_{sp1}}=c_{H1} \quad (11)$$

(2) From the material balance, compute $c_{C1}-x(=c_{C1}-c_{C1_{sp1}})$. Then compute $T_2$ from the energy balance, Eq. (9).

(3) Check whether $T_2 \leq T_{iH2}-\Delta T_m$. If so, go to step 4. Otherwise, go to step 6.

(4) Compute $c_{C1_{sp2}}$

$$C_{c1_{sp2}}=\frac{Q_2}{T_2-T_4} \quad (12)$$

(5) Terminate the process after calculating x using the results of step 2 and 4.

(6) Since the maximum value of $T_2$ is $T_{iH2}-\Delta T_m$, $T_2$ cannot be increased for the given $T_{iH}$. Therefore there is no way to increase $T_2$ without splitting hot stream H2. If the split ratio of stream H2 is determined by the method above, go to step 3 with increased $T_{iH2}$. Otherwise, the network of the combined MNU matrix is infeasible for the prescribed $\Delta T_m$.

The above network structure is applicable only if the target temperature of hot stream H1 is higher than the inlet temperature of cold stream C1 ($T_4$) plus $\Delta T_m$. If they are exactly the same, only splitting of C1 is required, allowing $T_3$ to be equal to $T_4$. Then, the stream H1 and the split portion of stream C1 are matched perfectly. If the target temperature of the hot stream H1 is lower than inlet temperature of cold stream C1 plus $\Delta T_m$, $T_3$ becomes $T_4$ with $x=0$, which is equivalent to just splitting of C1 without merging or bypass. The target temperature of H1 becomes just $T_4+\Delta T_m$ resulting in an imperfect match, which will require one extra unit.

It should be noted that, as a special case, if the inlet temperature of hot stream H2 is the pinch temperature and streams H1 and C1 belong to group A, then $T_2$ is easily calculated from Eq. (9) with Eq. (11).

$$c_{H1}(T_1-T_2)=c_{C1}(T_c-T_2) \quad (13)$$

Since this equation represents the energy balance above the pinch, $T_2$ should be equal to $T^*$. Then from Eq. (10) and the material balance, $x=c_{H1}-(c_{C1}-c_{C1_{sp2}})$, the following quantity can be first computed.

$$c_{C1}-c_{C1_{sp2}}=\frac{c_{H1}(T^*-T_3)}{T^*-T_4}=\frac{HC_{H1,BP}}{T^*-T_4} \quad (14)$$

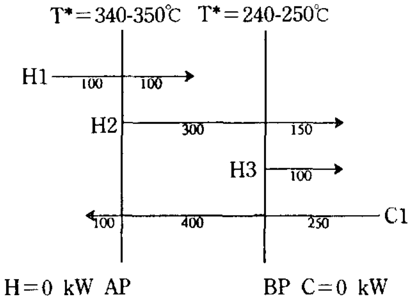
Finally x is calculated from the material balance

$$x=c_{H1}-(c_{C1}-c_{C1_{sp2}}) \quad (15)$$

**Table 3. Stream data for illustration 3**

| Stream | $T_i$(℃) | $T_t$(℃) | c(kW/℃) | HC(kW) |
|---|---|---|---|---|
| H1 | 400 | 300 | 2.0 | 200. |
| H2 | 350 | 200 | 3.0 | 450. |
| H3 | 250 | 200 | 2.0 | 100. |
| C1 | 177.5 | 365 | 4.0 | 750. |

$\Delta T_m = 10$℃




### Illustration 3

For the pinched problem of Table 3 [7], two pinch points are identified at T=240-250℃ and 340-350℃ and the enthalpy of each stream is also listed. For this problem, a split-merge network structure is synthesized as shown in Fig. 3. Since the inlet temperatures of the hot streams H2 and H3 are at the pinch temperatures, the split flow rates of the cold stream C1 are easily computed from Eq. (14) as 1 kW/℃ = 100 kW/(340-240)℃ and 2.4 kW/℃ = 150 kW/(240-177.5)℃, as given in Fig. 3.

## INITIAL MNU NETWORK AND ITS EVOLUTION

For pinched problems, two MNU/MER synthesis matrices are first obtained for the unpinched sub-problems above and below the pinch by the method proposed in the published paper [1]. To reduce the number of units in the network, instead of synthesizing each sub-problem independently, it is better for must-matches at the pinch to appear, if possible, in the other subnetwork. That means some matches selected at the pinch are determined by the must-matches of the other subnetwork.

Under the split-merge synthesis technique, some streams whose inlet and target temperatures enclose the pinch point should be matched with each other in both sub-networks. Since Eqs. (2) and (3) are satisfied simultaneously for those streams, streams of higher heat capacity flow rates should be split into branches which have exactly the same heat capacity flow rates of the corresponding opposite streams. These

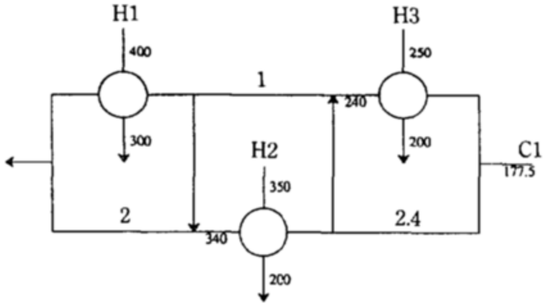



**Fig. 3. Synthesized network structure of illustration 3.**

streams satisfy the matching rule of Proposition 1 and thus will reduce the number of units in the network.

For the problem of Illustration 1, H2, H4 and C3 enclose the pinch point within their temperature ranges. Therefore C3 is split into three branches with heat capacity flow rates of 8.44 kW/℃ for H2, 7.0 kW/℃ for H4 and remaining 2.56 kW/℃ (=18.−8.44 −7.). Since $T_{tH2}$ and $T_{tH4}$ are lower than 150℃ (=140 +10), only splitting of stream C3 is required without merging or bypass and the number of units is not reduced.

If the initial network is not optimum, a better network can be sought by applying the three evolutionary phases proposed in the published paper [1]. First, we introduce a new unit by placing it before the largest unit identified using the Decision Index (DI). To improve the network further, we apply the H/2H rule (match between hottest stream and cold stream with second highest target temperature) to the largest DI unit or split the stream involved in that unit. In this step, heat loads cannot be redistributed along any loop consisting of units in AP and BP because this would result in heat transfer across the pinch point. That means that if heat loads are to be reassigned among units in a loop, those units must exist only in either AP or BP. When a loop is formed with units common to both sides of the pinch by introducing some new unit, the redistribution of heat loads must be restricted so that the following condition is met for any one of the hot or cold streams of the common units.

**Proposition 4. Discrete Heat Loads**

For a loop of units in BP, the redistribution of the heat loads of the common units which exist in both AP and BP is restricted to discrete values which allow the following condition to be satisfied for any one of the hot or cold streams.

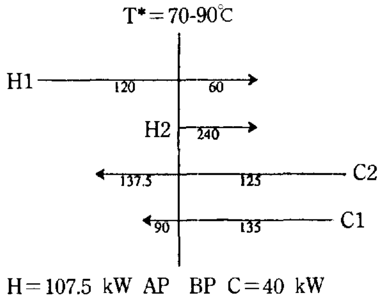
$$Q_{HE,AP} = HC_{AP} - \Sigma Q_{other\ match\ in\ AP} \quad \text{for common units} \tag{16}$$

For the proof of Proposition 4, the heat loads of

**Table 4. Stream data for illustration 4**

| Stream | $T_s$(℃) | $T_t$(℃) | c(kW/℃) | HC(kW) |
|---|---|---|---|---|
| H1 | 150 | 60 | 2.0 | 180. |
| H2 | 90 | 60 | 8.0 | 240. |
| C2 | 20 | 125 | 2.5 | 262.5 |
| C1 | 25 | 100 | 3.0 | 225. |

$\Delta T_m = 20$℃



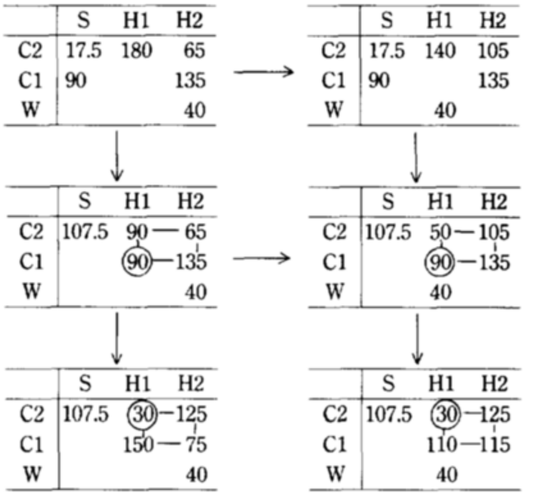




**Fig. 4. All feasible synthesis matrices of illustration 4 problem.**

the common units which exist in both AP and BP are the sums of the heat loads of those units in each sub-network. Since heat loads can be redistributed in each sub-network, the heat loads of the common units are expressed as

$$Q_{HE} = Q_{HE,AP} + Q_{HE,BP} \text{ for common units} \tag{17}$$

For any hot or cold streams, energy balance in AP is

$$HC_{AP} = (\Sigma Q_{HE})_{AP} \tag{18}$$

That is

$$HC_{AP} = Q_{HE,\,common,\,AP} + \Sigma Q_{other\ match,\,AP} \tag{19}$$

If a heat load loop exists in BP, $Q_{HE}$ of Eq. (17) can be varied in the loop by heat load redistribution. But, the value of $Q_{HE,AP}$ is constrained by Eq. (19). Therefore, the following equation must be satisfied for the common units even though a heat load loop exists in BP.

$$Q_{HE,AP} = HC_{AP} - \Sigma Q_{other\ match,\,AP} \tag{20}$$

**Illustration 4**

For the problem [7], for which the data are given in Table 4, an initial network is synthesized with H=107.5 kW, C=40 kW and T*=70-90℃ as shown in Fig. 4. Three other adjacent networks are also found by the enumeration method of the published paper [1]. In order to satisfy Eq. (16) for the heat load loop in BP, the heat loads of matches H1-C2 and H1-C1 can only be reassigned, with the values 30 kW (=137.5-107.5) for C2 or 90 kW (=90-0) for C1. However, for the heat load loop in AP and BP, the heat loads of matches S-C2, S-C1, H2-C1, and H2-C2 cannot be redistributed.

The synthesis procedure for a pinched problem is therefore summarized as follows:

1. Compute the minimum heating and cooling requirements and locate the pinch point. After dividing the problem into two sub-problems, AP and BP, at the pinch, find the must-matches in each subnetwork.
2. Search for the perfect matches in each subnetwork.
3. Choose the streams of group A to be matched with each other using the sufficient conditions of Proposition 1.
4. Determine the split ratios at the pinch to satisfy Eqs. (2) and (3).
5. Synthesize the other matches for each remaining unpinched sub-problem by using the H/H rule and the sufficient condition for MNU.
6. Find a better network using the three evolutionary phases of the published paper [1].

## APPLICATIONS

The effectiveness of proposed method is illustrated with a couple of literature application problems.

**1. Example 1**

An example from the literature, with the stream data given in Table 2, will be employed for comparative purpose [2, 11]. The pre-analysis results yield:

H=20 kW
C=60 kW

AP

| | S | H1 | H2 |
|---|---|---|---|
| C2 | | 240 | |
| C1 | 20 | | 90 |

BP

| | H1 | H2 |
|---|---|---|
| C2 | 90 | 30 |
| W | | 60 |

AP+BP

| | S | H1 | H2 |
|---|---|---|---|
| C2 | | 240 | |
| C1 | 20 | 90 | 120 |
| W | | | 60 |

**Fig. 5. Synthesis matrices of the example problem 1.**

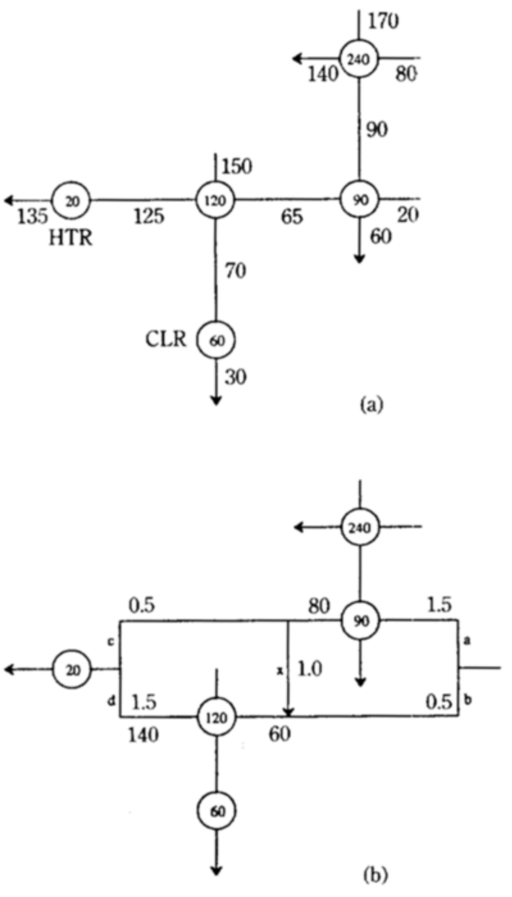



**Fig. 6. Network configurations of the example problem 1.**

$T^* = 80\text{-}90$℃

Must-matches = H1-C2 (rule 10 in AP), H2-C1 (rule 10 in AP and BP), H1-C1 (rule 10 in BP)

The heat loads of the synthesis matrices of the two sub-problems, above and below the pinch, and their combined MNU matrix are shown in Fig. 5. The net-

AP

| | S | H1 | H2 |
|---|---|---|---|
| C2 | 20 | 220 | |
| C1 | | 20 | 90 |

BP

| | H1 | H2 |
|---|---|---|
| C1 | 90 | 30 |
| W | | 60 |

AP+BP

| | S | H1 | H2 |
|---|---|---|---|
| C2 | 20 | 220 | |
| C1 | | 110 | 120 |
| W | | | 60 |

**Fig. 7. Synthesis matrices of an adjacent network.**

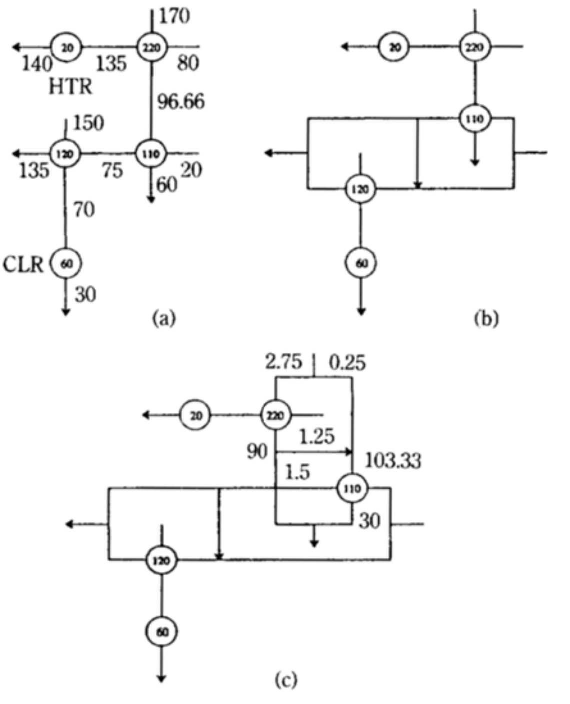



**Fig. 8. Network configurations of an adjacent network.**

work configuration of the combined MNU matrix is shown in Fig. 6(a). The unit of the H2-C1 match violates $\Delta T_m$ because $Q_{HE} > Q_{HE,AP}$. Thus splitting of stream C1 is required for MNU because the flow rate of stream C1 is the larger. With split streams a, b, c and d in Fig. 6(b), first $c_d$ is set to 1.5 kW/℃ from Eq. (11). With $c_c$ of 0.5 kW/℃ from the material balance, we can compute $T_c$ of 80 (=90-10) from Eq. (9). Since $c_d$ is 1.5 kW/℃ (=2-0.5), it satisfies the heat balance equation. Therefore $c_a$ is 1.5 kW/℃ and x (split ratio) is 1 kW/℃ (=1.5-0.5). Thus we can synthesize a feasible network from the combined MNU matrix without violating the prescribed $\Delta T_m$ just by introducing

AP

| | S | H1 | H2 |
|---|---|---|---|
| C2 | | 240 | |
| C1 | 20 | | 90 |

BP

| | H1 | H2 |
|---|---|---|
| C1 | 30 | 90 |
| W | | 60 |

AP+BP

| | S | H1 | H2 |
|---|---|---|---|
| C2 | | 240 | |
| C1 | 20 | 30 | 180 |
| W | | 60 | |

**Fig. 9. Synthesis matrices of another adjacent network.**

stream splitting.

By contrast, for the adjacent network shown in Fig. 7, which is computed using the method proposed in the published paper [1], it is impossible to obtain a feasible MER/MNU network. For the infeasible network structure without a split stream in Fig. 8(a), we can introduce a stream split to make it feasible. First $c_d$ is 1.5 kW/℃ from Eq. (11). With $c_c$ kW/℃ of 0.5 (=2-1.5), $T_c$ is 120℃ from Eq. (9). Since this temperature is greater than 86.66℃ (=96.66-10), we must increase the temperature of the hot stream by splitting it [Fig. 8(c)]. Since $T_c$ is still greater than the increased temperature of 93.33℃, there is no feasible network for this combined MNU matrix.

However, for an initial MNU/MER network which is still another adjacent network, from the must-match information, H2-C1 appears in both sub-networks. This match also satisfies the sufficient conditions for reducing the number of units for pinched problems. Therefore, H2 should be first matched with a split portion of C1 [$c_{C1_{sp1}}$ must be equal to $c_{H2}$ to satisfy both Eq. (2) and (3)]. Since H1-C2 is a perfect match in AP, an MNU/MER network is synthesized with H2-C1 and H1-C2. This corresponds to the modification of the BP synthesis matrix in Fig. 5 to the BP synthesis matrix in Fig. 9. Thus, a network is synthesized as shown in Fig. 10 without merging or bypass. Even though four MNU synthesis matrices are possible as shown in Fig. 11 (2 in AP and 2 in BP), only the MNU network of Fig. 8 is infeasible for this problem [11].

From an economical point of view, a network obtained using a reduced $\Delta T_m$ can be often more favorable for pinched problems. By reducing $\Delta T_m$, the utility cost decreases while the equipment cost increases. Consequently, the annual operating cost may be less than that of the MER/MNU network resulting from the prescribed $\Delta T_m$. In the example, the logarithmic mean temperature differences of the H1-C1 and H2-C1 matches of the MER/MNU network in Fig. 6(b) are 10℃ and 21.64℃, respectively. But, if $\Delta T_m$ is reduced from 10℃ to 5℃, those values become 12.427℃ and 31.915℃ for the network without any split stream shown in Fig. 6(a). This design turns out to be more favorable economically. Therefore to guarantee the optimum network we should find all the feasible networks using the above approaches: reduce $\Delta T_m$ and introduce stream splitting.

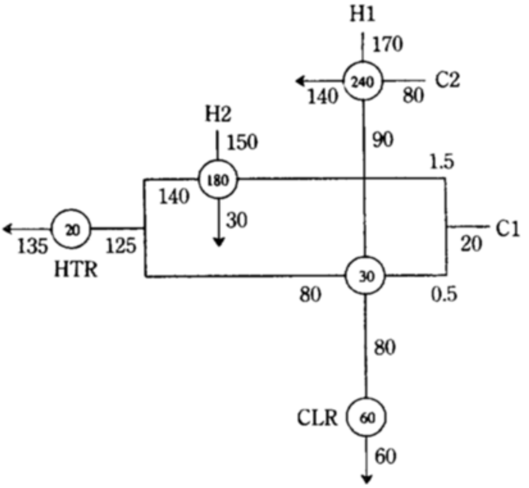



**Fig. 10. Network structure of another adjacent network.**

| | S | H1 | H2 |
|---|---|---|---|
| C2 | | 240 | |
| C1 | 20 | 30 | 180 |
| W | | 60 | |

| | S | H1 | H2 |
|---|---|---|---|
| C2 | | 240 | |
| C1 | 20 | 90 | 120 |
| W | | | 60 |

| | S | H1 | H2 |
|---|---|---|---|
| C2 | 20 | 220 | |
| C1 | | 50 | 180 |
| W | | 60 | |

| | S | H1 | H2 |
|---|---|---|---|
| C2 | 20 | 220 | |
| C1 | | 110 | 120 |
| W | | | 60 |

**Fig. 11. All possible synthesis matrices of the example problem 1.**

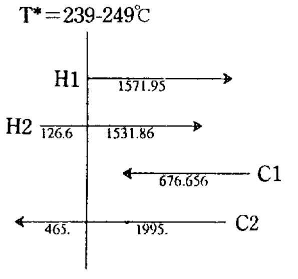
## 2. Example 2

For the problem shown in Table 5, solved by Floudas and Grossmann [9], the minimum heating and

**Table 5. Stream data for example problem 2**

| Stream | $T_i$(℃) | $T_f$(℃) | c(kW/℃) | HC(kW) |
|---|---|---|---|---|
| H1 | 249 | 100 | 10.550 | 1571.950 |
| H2 | 259 | 128 | 12.660 | 1658.460 |
| C1 | 96 | 170 | 9.144 | 676.656 |
| C2 | 106 | 270 | 15.000 | 2460.000 |

$\Delta T_m = 10$℃




H=338.4 kW AP BP C=432.154 kW

cooling requirements are first computed and then must-matches are found.

H=338.4 kW, C=432.154 kW, T*=239-249℃

We immediately establish the following matches:

S-C2 (rule 1 in AP, only one cold stream C2 exists in AP.)

H2-C2 (rule 1 in AP, only one cold stream C2 exists in AP.) or (rule 10 in AP, streams H2 and C2 pass through the pinch point of 239-249℃ in AP, while $C_{H2} < c_{C2}$ (12.66<15.).)

H1-C2 (rule 10 in BP, streams H1, H2 and C2 pass through the pinch temperature of 239-249℃ in BP. Match H2-C2 exists already. To satisfy Eq. (3) C2 must be split into two streams, one branch for H1 and the other for H2.)

H1-W (rule 4, $T_{fH1}$ (100) is lower than $T_{iC1} + \Delta T_m$ (96+10) of the coldest stream.)

Therefore, MNU=5 (=6-1)

Since there is no match for C1, the H1-C1 or H 2-C1 match should exist in the design. However, from Proposition 1, stream H2 has to be used for the H2-C2 match. Therefore the H1-C1 match is determined for the MNU network configuration as shown in Fig. 12. This network is the only network featuring the minimum number of units for this example.

Compared with Floudas and Grossmann's result [9] which is obtained using MILP method, this network has one less units. The network obtained in [9] is produced because the two sub-networks are synthesized independently at T*.

## CONCLUSIONS

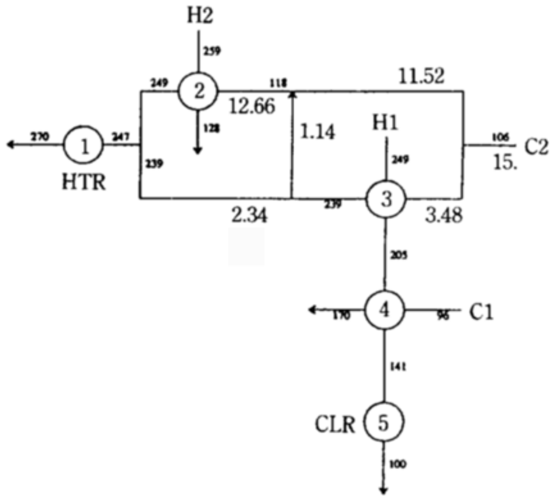



**Fig. 12. The MNU network configuration of the example problem 2.**

A new algorithmic-evolutionary approach for the systematic synthesis of pinched heat exchanger network is proposed. If the pinch point exists in a network, the problem is divided into two sub-problems which can be synthesized independently. If the initial MNU/MER network is obtained by applying to both unpinched sub-problems the H/H rule and the tick-off algorithm, then the sum of the number of units in the two sub-networks will in general be more than the theoretical MNU. Therefore, to guarantee MNU, a split-merge synthesis technique is used at the pinch point. A sufficient condition for MNU networks and quantitative calculation to determine network structures have been presented in detail. Tested against standard literature problems, this procedure proved to be efficient in finding feasible MNU/MER networks under pinch condition.

Since the split-merge synthesis method for pinched problems always requires a unit whose temperature difference is $\Delta T_m$, it may not be attractive economically because of the high cost of that unit. Therefore, from the economical point of view, all four suggested methods, namely, MNU relaxation, MER relaxation, $\Delta T_m$ relaxation and split-merge network structure, may need to be employed for network synthesis. Furthermore, since variation in $\Delta T_m$ causes variations in the annualized capital and utility costs of the network, the optimum value of $\Delta T_m$ has to be identified by an outer loop optimization.

## ACKNOWLEDGEMENT

The author would like to acknowledge the financial support of Honam Oil Refinery Co., Ltd.

## NOMENCLATURE

AP : sub-problem above the pinch point
BP : sub-problem below the pinch point
c : heat capacity flow rate [kW/℃]
C : cooling requirement [kW]
H : heating requirement [kW]
HC : heat content of stream [kW]
N : number of streams
n : number of streams
Q : amount of heat exchanged [kW]
S : steam
T : temperature [℃]
T* : pinch temperature [℃]
W : cooling water
x : split ratio [kW/℃]
$\Delta T_m$ : minimum allowable temperature approach [℃]

### Subscripts

AP : sub-problem above the pinch point
c : cold stream
h : hot stream
HE : heat exchanger
i : inlet condition
sp : split stream
t : target condition

## REFERENCES

1. Lee, I. and Reklaitis, G. V.: *Chem. Eng. Commun.*, **75**, 57 (1989).
2. Linnhoff, B. and Hindmarsh, E.: *Chem. Eng. Sci.*, **38**, 745 (1983).
3. Grimes, L. E., Rychener, M. D. and Westerberg, A. W.: *Chem. Eng. Commun.*, **14**, 339 (1982).
4. Linnhoff, B., Mason, D. R. and Wardle, I.: *Comput. Chem. Eng.*, **3**, 295 (1979).
5. Papoulias, S. A. and Grossmann, I. E.: *Comput. Chem. Eng.*, **7**, 707 (1983).
6. Su, J. and Motard, R. L.: *Comput. Chem. Eng.*, **8**, 67 (1984).
7. Wood, R. M., Wilcox, R. J. and Grossmann, I. E.: *Chem. Eng. Commun.*, **39**, 371 (1985).
8. Hohmann, E. C.: Ph.D. Thesis, University of Southern California (1971).
9. Floudas, C. A. and Grossmann, I. E.: *Comput. Chem. Eng.*, **10**, 153 (1986).
10. Douglas, J. M.: "Conceptual Design of Chemical Processes", McGraw-Hill, New York, NY (1988).
11. Jones, S. A. and Rippin, D. W. T.: Symposium Series No. 92, 157, PSE 85, Cambridge, England (1985).